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by

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ABSTRACT

This paper generalizes the Zacks model for minefield crossings. Zacks computes in his model the probability of the Nth vehicle crossing a minefield and also the distribution of the number of vehicles crossing the field. Zacks' computations are made under the assumption that all the vehicles are of the same type and only one kind of mine is present in the field. This paper removes both these restrictions.

THE GENERALIZED ZACKS MODEL

by

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The purpose of this note is to generalize the Zacks minefield model* for computing the probabilities of vehicles crossing a minefield. In the Zacks model only one type of mine and one type of vehicle are considered. This paper shows how the model can be extended to different mines and different vehicles. This note follows the Zacks paper closely, and the reader is advised to have a copy of that paper at hand when reading this note.

The Zacks model begins with the discussion of the number of mines in the path of the vehicle. In the Zacks model, only the number of mines is required; here, the number of mines for each mine type is needed.

The following notation is introduced:

H = mine type, H=1,2,...,M

N(H) = number of clusters of the Hth type mine

N(I,H) = number of the Hth type mines in the Ith cluster

J(H) = random number of the Hth type mines in the path of a given tank

J(I,H) = random number of the Hth type mines from the Ith cluster that are in the path of the tank

Then:

$$J(H) = \sum_{I=1}^{N(H)} J(I,H) \qquad \text{(Eq 1)}$$

*Zacks, S. Survival Distributions in Crossing Fields Containing Clusters of Absorption Points with Possible Detection and Uncertain Activation or Absorption. Technical Report 23, Department of Mathematics and Statistics, Case Western Reserve University, Cleveland, Ohio. June 1976.

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Now

$\binom{N}{M}$ = number of combinations of N things taken M at a time

$\psi(I,H)$ = *probability that one of the Hth type mines from the Ith cluster is in the path*

Then:

$Pr\{J(I,H)=j\}$ = probability that j mines, coming from the Ith cluster of the Hth type mine, are in the path

The probability that j of these mines are in the path is given by:

$$Pr\{J(I,H) = j\} = \binom{N(I,H)}{j} \bullet \psi(I,H)^j \bullet \{1-\psi(I,H)\}^{\{N(I,H)-j\}} \quad \text{(Eq 2)}$$

Summing over all clusters yields the probability that a total of j mines are in the path:

$$Pr\{J(H) = j\} = \sum_{j=j_1+j_2+\ldots+j_{N(H)}} \cdots \sum \prod_{k=1}^{N(H)} Pr\{J(k,H)=j_k\} \quad \text{(Eq 3)}$$

Equation 3 is Zacks' equation 3.9 with a subscript H for mine type. Note that $Pr\{J(1),\ldots,J(M)\}$ is the product of the $Pr\{J(H)\}$'s.

Now let:

$P_d(H,V)$ = probability that the Vth type vehicle will detect the Hth type mine

$P_a(H,V)$ = probability that the Vth type vehicle will activate the Hth type mine

$P_k(H,V)$ = probability that the Vth type vehicle will be killed by the Hth type mine

Then the probability that a vehicle V will survive an encounter with an Hth type mine is:

$$S(H,V) = P_d(H,V) + \{1-P_d(H,V)\} \bullet \left[\{1-P_a(H,V)\} + P_a(H,V) \bullet \{1-P_k(H,V)\}\right] \quad \text{(Eq 4)}$$

Hence, the probability that the first vehicle of type V_1 will survive the Hth type mine encounter, given J(H)=j, is given by equation 5:

$$Pr\{S_1(H,V_1) \mid J(H)=j\} = S(H,V_1)^j \qquad \text{(Eq 5)}$$

The probability that the first vehicle will survive, given the J's, is the product $Pr\{S_1(V)\text{'s} \mid J\text{'s}\}$ over all mine types. Hence, the probability of survival for the first vehicle is the sum over the J's of the product (the probability of survival given the J's and the probability of the J's); that is:

$$Pr\{S_1(V_1)\} = \prod_{H=1}^{M} \sum_{j(H)=0}^{N(H)} Pr\{S_1(H,V_1) \mid J(H) = j(H)\} \bullet Pr\{J(H) = j(H)\}$$

where M is the number of mine types. This corresponds to Zacks' equation 4.3.

Additional notation is required to calculate the probability that the second vehicle V_2 crosses the minefield:

Let:

$M_1(H,V)$ = *random number* of Hth type mines destroyed by the Vth type vehicle on a crossing of the path

$Pr\{M_1(1,V)=m_1, M_1(2,V)=m_2 \ldots M_1(M,V)=m_M \mid J(1)=j_1 \ldots J(M)=j_M\}$ = probability that m_1 of the first type mine, m_2 of the second type mine,... m_M of the last type mine are destroyed, given that j_1 of the first type mine, j_2 of the second type mine,... j_M of the last type mine are in the path

$$W(H,V) = \{1-P_d(H,V)\} \; \{1-P_a(H,V)\} \qquad \text{(Eq 6)}$$

$Pr\{S_2 \mid M_1(1,V)=m_1 \ldots M_1(M,V)=m_M, J(1)=j_1 \ldots J(M)=j_M\}$ = probability that the second vehicle survives, given M_H of J_H *mines in the path have been destroyed*

Thus:

$$\Pr\left\{S_2 \middle| M_1(1,V)=m_1 \ldots M_1(M,V)=m_M, J(1)=j_1 \ldots J(M)=j_M\right\} = \prod_{H=1}^{M} S(H,V)^{j_H - m_H}$$

And:

(Eq 7)

$$\Pr\left\{M_1(1,V)=m_1 \ldots M_1(M,V)=m_M \middle| J(1)=j_1 \ldots J(M)=j_M\right\} = \prod_{H=1}^{M} \binom{j_H}{m_H} \bullet \quad \text{(Eq 8)}$$

$$\left\{S(H,V)-W(H,V)\right\}^{m_H} W(H,V)^{j_H-m_H} + \sum_{H=1}^{M} \left\{1-S(H,V)\right\} \left\{S(H,V)-W(H,V)\right\}^{m_H-1} \bullet$$

$$\sum_{r_H=m_H-1}^{j_H-1} \binom{r_H}{m_H-1} W(H,V)^{r_H-m_H+1} \prod_{i \neq H} \left\{S(i,V)-W(i,V)\right\}^{m_i} \sum_{r_i=m_i}^{j_i} \binom{r_i}{m_i} W(i,j)^{r_i-m_i}$$

This is Zacks' equation 4.16; hence, the probability that the second vehicle survives is given by:

$$\Pr\{S_2\} = \sum_{j_1=0}^{N(1)} \cdots \sum_{j_M=0}^{N(M)} \sum_{m_1=0}^{j_1} \cdots \sum_{m_M=0}^{j_M} \Pr\left\{M_1(1,V)=m_1 \ldots\right.$$

(Eq 9)

$$\left. M_1(M,V)=m_M \middle| J(1)=j_1 \ldots J(M)=j_M \right\} \prod_{H=1}^{M} \Pr\left\{J(H)=j_H\right\} S(H,V)^{j_H-m_H}$$

The calculation of the probability that the nth vehicle V_n survives is obtained by considering the probability that the vehicle survives an Hth type mine encounter, given that there were originally j(H) of such mines and m_{n-1} were destroyed by the n-1 vehicles that preceded the current vehicle.

Let:

$M_{n-1}(H,V_{n-1})$ = random number of Hth type mines destroyed by the passing of the V_{n-1} vehicles over the path

$S_n(H,V_n)$ = the random event of the nth vehicle surviving the Hth type mine

Then:

$$Pr\left\{S_n(V_n) \middle| M_{n-1}(1,V_{n-1})=m_1 \ldots M_{n-1}(M,V_{n-1}) \quad J(1)=j_1 \ldots J(M)=j_M\right\} =$$

probability that the nth vehicle survives given that m_H of the j_H

mines have been destroyed $= \prod_{H=1}^{M} S(H,V_n)^{j_H-m_H}$ (Eq 10)

Thus:

$$Pr\left\{S_n(V_n), M_{n-1}(1,V_n)=m_1 \ldots M_{n-1}(H,V_n)=m_H \middle| J(1)=j_1 \ldots J(M)=j_M\right\} =$$

$$Pr\left\{M_{n-1}(1,V_n)=m_1 \ldots M_{n-1}(H,V_n) \middle| J(1)=j_1 \ldots J(M)=j_M\right\} \prod_{H=1}^{M} S(H,V_n)^{j_H-m_H}$$

(Eq 11)

Now, the probability of m_n losses having occurred with the V_n crossing by the Hth type mine, given that there had been j mines in the path, must be calculated.

Let:

$M_n(H,V_n)$ = random number of Hth type mines destroyed by the first n vehicle crossings.

These probabilities will be calculated recursively as follows:

$$\Pr\left\{M_n(1,V)=m_1 \ldots M_n(M,V)=m_M \mid J(1)=j_1 \ldots J(H)=j_H\right\} =$$

(Eq 12)

$$\sum_{r_1=0}^{m_1} \cdots \sum_{r_M=0}^{m_M} \Pr\left\{M_1(1,V_{m-1})=r_1 \ldots M_1(M,V_{n-1})=r_H \mid J(1)=j_1-m_1+r_1\right\} \bullet$$

$$\Pr\left\{M_{n-1}(1,V_n)=M_1-r_1 \ldots M_{n-1}(M,V_{n-1})=m_M-r_M \mid J(1)=j_1 \ldots J(M)=j_M\right\}$$

This probability depends on the order in which vehicles enter the path. Now the probability that the nth vehicles survives is given by:

$$\Pr\left\{S_n(V_n) \mid J(1)=j_1 \ldots J(H)=j_H\right\} \bullet \sum_{m_1=0}^{j_1} \cdots \sum_{m_M=0}^{j_M} \Pr\left\{M_{n-1}(1,V_{n-1})=m_1 \ldots\right.$$

(Eq 13)

$$\left. M_{n-1}(M,V_{n-1})=m_M \mid J(1)=j_1 \ldots J(M)=j_M\right\} \prod_{H=1}^{M} S(H,V_n)^{j_H-m_H}$$

Hence, the probability that V_n survives, given the J's, is:

$$\Pr\{S_n\} = \sum_{j_1=0}^{N(1)} \cdots \sum_{j_M=0}^{N(M)} \Pr\left\{S_n(V_n) \mid J(1)=j_1 \ldots J(M)=j_M\right\} \prod_{H=1}^{M} \Pr\{J(H)=j_H\}$$

(Eq 14)

In conclusion, the paper considers the distribution of the number of survivors of N crossings. This is determined recursively.

Let:

$I(K)$ = a random variable; 1, if the Kth vehicle survives the minefield crossing; 0, otherwise

$X(K)$ = a random number of vehicles out of K that have survived crossing the minefield.

$$X(K) = \sum_{h=1}^{K} I(h) \qquad \text{(Eq 15)}$$

Now the joint probability of X_1 and the $M_1(1,V_1)\ldots M_1(M,V_1)$ given $J(1),\ldots J(M)$ is given as follows:

$$\Pr\{X(1)=1, M_1(1,V_1)=m_1,\ldots M_1(M,V_1)=m_M \mid J(1)=j_1,\ldots J(M)=j_M\}$$

$$= \prod_{H=1}^{M} \binom{j_H}{m_H} W(H,V_1)^{m_H} \{S(H,V_1)-W(H,V_1)\}^{j_H-m_H} \qquad \text{(Eq 16)}$$

Hence:

$$\Pr\{X(1)=0, M_1(1,V_1)=m_1,\ldots M_1(M,V_1)=m_M \mid J(1)=j_1,\ldots J(M)=j_M\}$$

$$=\Pr\{M_1(1,V_1)=m_1,\ldots M_1(M,V_1)=m_M \mid J(1)=j_1,\ldots J(M)=j_M\} -$$

$$-\Pr\{X(1)=1, M_1(1,V_1)=m_1,\ldots M_1(M,V_1)=m_M \mid J(1)=j_1,\ldots J(M)=j_M\}$$

(Eq 17)

Now the recursive probabilities for $X(k)$ and $M_k(1,V_k)\ldots M_k(M,V_k)$ given $J(1)\ldots J(M)$ are written as:

$$Pr\left\{X(K)=i, M_k(1,V_k)=M_1, \ldots M_k(M,V_k) \mid J(1)=j_1 \ldots J(M)=j_M\right\}$$

$$= \sum_{r_1=0}^{m_1} \ldots \sum_{r_M=0}^{m_M} \Big[Pr\left\{X_{k-1}=i-1, M_{k-1}(1,V_{k-1})=r_1 \ldots M_{k-1}(M,V_{k-1})=r_m \mid J(1)=j_1 \ldots J(M)=j_M\right\} \bullet$$

$$\bullet\ Pr\left\{X(1)=1, M_1(1,V_k)=m_1-r_1 \ldots M_1(M,V_k)=m_M-r_M \mid J(1)=j_1-r_1 \ldots J(M)=j_M\right\} +$$

$$+\ Pr\left\{X_{k-1}=i, M_{k-1}(1,V_{k-1})=r_1 \ldots M_{k-1}(M,V_{k-1})=r_m \mid J(1)=j_1 \ldots J(M)=j_M\right\} \bullet$$

$$\bullet\ Pr\left\{X(1)=0, M_1(1,V_k)=m_1-r_1 \ldots M_1(M,V_k)=m_M-r_M \mid J(1)=j_1-r_1 \ldots J(M)=j_M\right\}\Big]$$

(Eq 18)

Then:

$$Pr\left\{X(K)=i\right\} = \sum_{j_1=0}^{N(1)} \ldots \sum_{j_H=0}^{N(H)} \sum_{M_1=0}^{j_1} \ldots \sum_{M_H=0}^{j_H} \bullet$$

$$\bullet\ Pr\left\{X(K)=i, M_K(1,V_K)=M_1, \ldots, M_K(H,V_K)=M_H \mid J(1)=j_1, \ldots J(H)=j_H\right\}$$

(Eq 19)

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